A Multigrid Strongly Implicit Procedure for Transonic Potential Flow Problems

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Abstract

THE transonic full potential equation in strong conservation form is solved on a body-fitted coordinate system using a strongly implicit procedure enhanced by the multigrid procedure. Weak viscous effects are accounted for by computing the displacement thickness using the Nash-McDonald integral boundary-layer equation. The convergence speed of the multigrid procedure is found to be comparable to the state-of-the-art multigrid-alternating direction implicit procedures on identical grids. Several numerical cases, some involving weak viscous effects, are presented and compared with published results.

Contents

Recently the author and his co-workers developed a relaxation procedure for the solution of the steady transonic full potential equation in a body-fitted coordinate system.[1] This procedure has the following steps. The governing equation is linearized at each step of the iteration by lagging the density by one iteration. The density is biased in supersonic regions using the artificial compressibility concept.[2,3] Standard central differences are used to discretize the governing equation and the transformation metrics. The above steps lead to the following system of equations:

$$[M]^n\{c\}^{n+1}=\{R\}^n \tag{1}$$

where $[M]$ is a sparse, pentadiagonal coefficient matrix; c^{n+1} the change in the velocity potential ϕ between successive iteration levels $(n+1)$ and n; R^n the residual of the discretized governing equation.

In Ref. 1, Stone's strongly implicit procedure (SIP)[4] was used to replace $[M]$ with a new set of matrices $[N]$ that are close to $[M]$ but are easily inverted. Individual members of this set of matrices differ from each other only in terms of a relaxation parameter α, cyclically varied from 0 to 1. Stone[4] recommends the cyclical variation in order to damp out all the components of the error c^{n+1} quickly.

In the present work the above procedure is modified by the use of the multigrid procedure. Rather than varying α to generate several N matrices, the problem is transferred to a series of coarse grids, each being a subset of the original fine grid. Each grid is ideally suited to damp out one band of error components in c^{n+1}. Thus, the net effect is the same as cycling the α parameter. Considerable efficiency is gained however, because of the lower operation counts in the coarse grids. Since the problem is nonlinear, the "full approximate scheme" version of the multigrid procedure is used.

Brandt has given extensive discussions of the multigrid process in literature.[5] For transonic flows, several workers have combined the SLOR, ADI, and other relaxation procedures with multigrid techniques.[6,7] Because the multigrid process is well documented, a discussion of the multigrid-SIP approach is not presented. Details of the implementation, such as the treatment of the nonlinearity, grid transfer operators, data base organization, choice of relaxation factors, etc., are given in Ref. 8. Here we describe only the performance of the multigrid-SIP approach for a set of test cases.

As a first test case, Poisson's equation with Dirichlet boundary conditions on a uniformly-spaced rectangular grid was studied. A 96×64 fine grid and five coarse grids were used. An error reduction rate of 0.5 was achieved when four grids were used. Increasing the total number of grids further showed no noticeable improvement.

In the preceding case, as well as in all subsequent cases, the error reduction rate is defined as $(R^n_{\max}/R^0_{\max})^{1/n}$, where $R_{\max}$ is the largest absolute residual and n is the total equivalent fine grid work. All residual calculations in every grid were accounted for in the work, regardless of whether these residuals were used in calculations on that grid or being transferred to a coarser grid.

As a second application, the problem of nonlifting subsonic and transonic flow past thick, sharp-nosed airfoils was considered. A sheared Cartesian coordinate system, with a high degree of stretching in both coordinate directions, was chosen. A 136×24 fine grid and three coarse grids were used. Numerical calculations for a 10% parabolic arc airfoil at subsonic and transonic Mach numbers showed that the multigrid-SIP approach converges to the "correct" numerical solution. In Fig. 1, the residual variation vs equivalent fine grid work is shown when two, three, and four grids are used

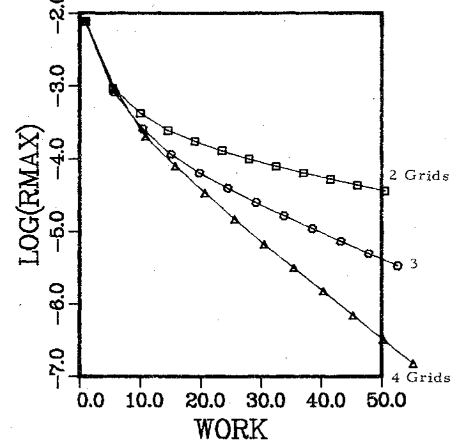

Fig. 1 Convergence history of the multigrid-SIP procedure for the subsonic potential flow past a 10% circular arc airfoil on a stretched grid.

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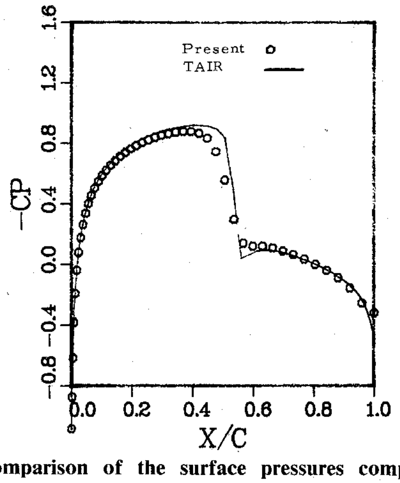

Fig. 2 Comparison of the surface pressures computed by the multigrid-SIP and the TAIR programs. NACA 0012 airfoil, Mach number 0.8, zero angle of attack.

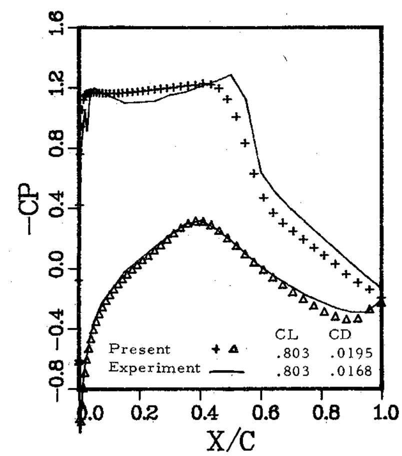

Fig. 3 Multigrid-SIP calculations involving weak viscous effects. (RAE 2822 airfoil, 0.73 Mach number, 6.5×10^6 Reynolds number, 3.19 deg angle of attack.)

for the Mach number 0.72. When four grids were used an error reduction rate of 0.8 was achieved, and it was possible to reduce residuals to double precision round off accuracy on a VAX 11/780 machine. At higher Mach numbers the reduction rate was somewhat less impressive, although no convergence difficulties were encountered.

In order to compare the performance of the multigrid-SIP approach with state-of-the-art ADI procedures, a series of airfoil calculations were performed on a sheared parabolic grid with a 128×32 fine grid. The grid stretching was identical to that used in FLO43, a multigrid-ADI-finite volume full potential code.[9] Three coarse grids, in addition to the above fine grid, were used. Both the codes were executed in a "hands-off" manner, without any attempt to fine tune the relaxation factors. The multigrid-SIP approach was found to be comparable to ADI procedures based on several test cases. For nonlifting subsonic flows an error reduction rate of 0.782 was achieved with the multigrid-SIP approach, while lifting moderately strong shock cases lead to reduction rates of 0.9. While the present comparison with ADI procedures is not conclusive, it does indicate that the multigrid-SIP approach can be competitive. The reader is referred to Ref. 1 for other issues, such as the operation counts required by the SIP and ADI procedures.

For moderate and strong shock cases, convergence difficulties were encountered unless the artificial viscosity terms were turned on at points with Mach numbers well below 1. This, as expected, led to some shock smearing. Any attempt to reduce the artificial viscosity led to a periodic build up and decay of overshoots ahead of the shock, and a consequent deterioration in convergence. At this writing no satisfactory solution to this problem has been found by the author. A plot of 160×32 grid results for the NACA 0012 airfoil at zero angle of attack and Mach number 0.80 is shown in Fig. 2 and compared with the results of the TAIR program.[2] Note that the high artificial viscosity tends to smear the re-expansion at the foot of the shock.

Finally, a series of calculations was performed that included viscous effects on a 160×32 fine grid with three coarse grids. The displacement thickness was computed during each cycle just before the fine grid relaxation using the Nash-McDonald integral boundary-layer procedure. The boundary-layer growth was then simulated on the fine grid, as well as the coarse grids, using a transpiration velocity distribution on the airfoil surface. The multigrid process converged smoothly despite the fact that the surface transpiration velocity was being changed constantly. In Fig. 3 the surface pressure distribution for an RAE 2822 airfoil is compared with experiments at Mach number 0.73 and 3.19 deg angle of attack and satisfactory agreement is observed. In Ref. 8 the residual reduction history for this case as well as other viscous-inviscid interaction cases may be found. For these calculations approximately 200 fine grid work units were needed to reduce residuals to below 10^{-6}.

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